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NO CONSISTENT CROSS-INTERACTIONS FOR A COLLECTION OF MASSLESS SPIN-2 FIELDS

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We report a no-go theorem excluding consistent cross-couplings for a collection of massless, spin-2 fields described, in the free limit, by the sum of Pauli-Fierz actions (one for each field). We show that, in spacetime dimensions > 2, there is no consistent coupling, with at most two derivatives of the fields, that can mix the various "gravitons". The only possible deformations are given by the sum of individual Einstein-Hilbert actions (one for each field) with cosmological terms. Our approach is based on the BRST-based deformation point of view[†].

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1 Introduction

One striking feature of the Einstein theory of gravity is that it involves a single massless spin-two field. We report here results obtained recently [1] showing that, in fact, this is not an accident : theories involving different types of gravitons with non trivial, consistent, cross-interactions simply do not exist. This no-go theorem holds under the assumption that (i) the Lagrangian contains no more than two derivatives of the massless spin-2 fields $\{h^a_{\mu\nu}\}$ ($a = 1, \cdots, N$); (ii) the interactions can be continuously switched on; and (iii) in the limit of no interaction, the action reduces to the sum of one Pauli-Fierz action [2] for each field $h^a_{\mu\nu}$, i.e.

$$\begin{aligned} S_0[h^a_{\mu\nu}] \;=\; & \sum_{a=1}^{N} \int d^n x \left[-\frac{1}{2} \left(\partial_\mu h^a{}_{\nu\rho}\right) \left(\partial^\mu h^{a\nu\rho}\right) + \left(\partial_\mu h^{a\mu}{}_\nu\right) \left(\partial_\rho h^{a\rho\nu}\right) \right. \\ & \left. - \left(\partial_\nu h^{a\mu}{}_\mu\right) \left(\partial_\rho h^{a\rho\nu}\right) + \frac{1}{2} \left(\partial_\mu h^{a\nu}{}_\nu\right) \left(\partial^\mu h^{a\rho}{}_\rho\right) \right] \end{aligned} \tag{1}$$

(spacetime indices are raised and lowered with the flat Minkowskian metric $\eta_{\mu\nu}$, for which we use a "mostly plus" signature).

The free action (1) is invariant under the linear gauge transformations, $\delta_\epsilon h^a_{\mu\nu} = \partial_\mu \epsilon^a_\nu + \partial_\nu \epsilon^a_\mu$ where the ϵ^a_ν are arbitrary functions. These transformations are abelian and irreducible. The Pauli-Fierz action is in fact the linearized Einstein action and describes a pure spin-2 system (no spin 1 or 0 included).

The equations of motion are $H^{\mu\nu}_a = 0$, where $H^a_{\mu\nu}$ is the linearized Einstein tensor. The Noether identities expressing the gauge invariance of the free action are $\partial_\nu H^{a\mu\nu} = 0$ (linearized Bianchi identities). The gauge symmetry removes unwanted unphysical states.

The problem of introducing consistent interactions for a collection of massless spin-2 fields is that of adding local interaction terms to the action (1) while modifying at the same time the original gauge symmetries if necessary, in such a way that the modified action is invariant under the modified gauge symmetries. Since we are interested in the classical theory, we shall also demand that the interactions contain no more than two derivatives so that the nature of the differential equations for $h^a_{\mu\nu}$ is unchanged. We shall, however, make no assumption on the polynomial order of the fields in the Lagrangian or in the gauge symmetries.

In an interesting work [3], Cutler and Wald have proposed multi-graviton theories with cross-interactions based on associative, commutative algebras. These authors arrived at these structures by focusing on the modified gauge transformations and their algebra, but did not analyze the extra conditions that must be imposed on the modified gauge symmetries if these are to be compatible with a Lagrangian having the free field limit prescribed above.

Some explicit examples of Lagrangians that realize the Cutler-Wald algebraic structures have been constructed in [4] and [5], but none of these has the correct free field limit. In fact, their free field limit does involve a sum of Pauli-Fierz Lagrangians, but some of the "gravitons" come with the wrong sign and thus, the energy of the theory is unbounded from below. To our knowledge, the question of whether other examples of (real) Lagrangians realizing the Cutler-Wald structure could exist and whether some of them would have the correct free field limit was left open.

Motivated by these developments, we have re-analyzed the question of consistent interactions for a collection of massless spin-2 fields by imposing from the outset that the deformed Lagrangian should have the free field limit (1). It turns out that this requirement forces one additional condition on the Cutler-Wald algebra defining the interaction, namely, that it should be symmetric in the scalar product defined by the free Lagrangian. This extra constraint is quite stringent and implies that the algebra is the direct sum of one-dimensional ideals. This eliminates all the cross-interactions. The only consistent deformation (within the context of no more than two derivatives) that the free theory based on (1) admits is the sum of one Einstein-Hilbert action (with a possible cosmological term) for each spin-two field,

$$S[g^a_{\mu\nu}] = \sum_a \frac{2}{\kappa_a^2} \int d^n x (R^a - 2\Lambda^a)\sqrt{-g^a},\ g^a_{\mu\nu} = \eta_{\mu\nu} + \kappa^a h^a_{\mu\nu} \tag{2}$$

where R^a is the scalar curvature of $g^a_{\mu\nu}$ and g^a its determinant. There is no other possibility. [Some sectors may remain undeformed; for $\kappa_a = 0$, the action reduces to the Pauli-Fierz term plus a possible cosmological term $\lambda^a h^{a\mu}_{\ \ \mu}$.]

We present here the main ideas underlying our no-go theorem. Details and proofs may be found in [1].

2 Cohomological reformulation

Our approach is based on the BRST reformulation of the problem [6], in which consistent couplings define deformations of the solution of the so-called "master equation". The advantage of this approach is that it clearly organizes the calculation of the non-trivial consistent couplings in terms of cohomologies which are known or easily computed. These cohomologies are in fact interesting in themselves, besides their occurence in the consistent interaction problem. The use of BRST techniques somewhat streamlines the derivation, which would otherwise be more cumbersome.

Let us thus write down first the solution of the master equation for a collection of free, spin-2, massless fields. According to the general rules [7], the spectrum of fields, ghosts and antifields is given by : (i) the fields $h^a_{\alpha\beta}$, with ghost number and antighost number zero; (ii) the ghosts C^a_α, with ghost number one and antighost number zero; (iii) the antifields $h^{*\alpha\beta}_a$, with ghost number minus one and antighost number one; and (iv) the antighosts $C^{*\alpha}_a$, with ghost number minus two and antighost number two.

While the ghost number assignments are rather standard, the introduction of another grading, namely, the antighost number, may appear to be a bit artificial. It turns out, however, that this is not so. The antighost number (also called antifield number) is not only technically useful, but it also enables one to keep track of terms with different meanings in the master equation. We shall come back to this point at the end of this section.

According to the prescriptions of [7], the solution of the master equation for the free theory is, $W_0 = S_0 + \int d^n x\, h^{*\alpha\beta}_a (\partial_\alpha C^a_\beta + \partial_\beta C^a_\alpha)$, from which we get the BRST differential s of the free theory as $s\cdot = (W_0, \cdot)$. Here, $(,)$ is the antibracket. Explicit calculations show that s splits as $s = \delta + \gamma$ where the action of γ and δ on the variables is zero except

$$\gamma h^a_{\alpha\beta} = 2\partial_{(\alpha} C^a_{\beta)},\ \delta h^{*\alpha\beta}_a = \frac{\delta S_0}{\delta h^a_{\alpha\beta}},\ \delta C^{*\alpha}_a = -2\partial_\beta h^{*\beta\alpha}_a. \tag{3}$$

The decomposition of s into δ plus γ is dictated by the antighost number: δ decreases the antighost number by one unit, while γ leaves it unchanged. One has $\delta^2 = 0,\ \delta\gamma + \gamma\delta = 0,\ \gamma^2 = 0$.

If one expands the solution W of the master equation $(W, W) = 0$ for the searched-for interacting theory in powers of the deformation parameter g (coupling constant), $W = W_0 + gW_1 + g^2 W_2 + \cdots$, one finds the conditions $sW_1 \equiv (W_0, W_1) = 0$ and $(W_1, W_1) = -2sW_2$ at orders one and two, respectively. The first condition expresses that the first-order deformation W_1 should be a BRST-cocycle. Trivial cocycles (of the form sK) define actually "fake" interactions, in the sense that they can be absorbed through fields and ghosts redefinitions [6]. The second condition expresses that (W_1, W_1) - which is easily verified to be BRST-closed - should be BRST-exact in order for W_2 to exist. Since we deal with local functionals, the relevant cohomology groups are, in terms of the integrands, $H(s|d)$ [8, 9]. Thus, first-order deformations are characterized by elements of $H^{0,n}(s|d)$ (BRST cohomology at ghost number zero and form degree n for the n-form integrand a of the ghost number functional $W_0 = \int a$); and obstructions to continuing a given first-order consistent deformation to order g^2 are measured by $H^{1,n}(s|d)$.

In the sequel, we shall compute explicitly $H^{0,n}(s|d)$ for a collection of free, massless spin-2 fields, i.e., we shall determine all possible first-order consistent interactions. We shall then determine the conditions that these must fulfill in order to be unobstructed at order g^2. These conditions turn out to be extremely strong and prevent cross interactions between the various types of gravitons.

We finally close this section by observing that W_0 and W have ghost number zero, but break into various components with different antighost numbers. For instance, W_0 has a piece with antighost number zero and another with antighost number one. The first piece is the classical action, while the second contains the information about the gauge symmetries. This feature is quite general: the antighost number zero component of the solution of the master equation is the classical action, the antighost number one component contains the information about the gauge symmetries while the antighost number two component contains the information about the gauge algebra. The absence of such a term in W_0 reflects the fact that the gauge algebra of the free theory is abelian. By deforming the solution of the master equation, one deforms everything (action, gauge transformations, gauge algebra) at once; but one can recover the detailed information by splitting W according to the antighost number.

3 Cohomology of γ

To compute the consistent, first order deformations, i.e., $H(s|d)$, we need $H(\gamma)$ and $H(\delta|d)$. We start with $H(\gamma)$, which is rather easy.

As it is clear from its definition, γ is related to the gauge transformations. Acting on anything, it gives zero, except when it acts on the spin-2 fields, on which it gives a gauge transformation with gauge parameters replaced by the ghosts.

The only gauge-invariant objects that one can construct out of the gauge fields $h^a_{\mu\nu}$ and their derivatives are the linearized curvatures $K^a_{\alpha\beta\mu\nu}$ and their derivatives. The antifields and their derivatives are also γ-closed. The ghosts and their derivatives are γ-closed as well but their symmetrized first order derivatives are γ-exact , as are all their subsequent derivatives since $\partial_{\alpha\beta} C^a_\gamma = \frac{1}{2}\gamma\left(\partial_\alpha h^a_{\beta\gamma} + \partial_\beta h^a_{\alpha\gamma} - \partial_\gamma h^a_{\alpha\beta}\right)$.

It follows straightforwardly from these observations that the γ-cohomology is generated by the linearized curvatures, the antifields and all their derivatives, as well as by the ghosts C^a_μ and their antisymmetrized first-order derivatives $\partial_{[\mu}C^a_{\nu]}$. More precisely, let $\{\omega^I\}$ be a basis of the space of polynomials in the C^a_μ and $\partial_{[\mu}C^a_{\nu]}$ (since these variables anticommute, this space is finite-dimensional). One has:

$$\gamma a = 0 \Rightarrow a = \alpha_J\left([K],[h^*],[C^*]\right)\omega^J\left(C^a_\mu, \partial_{[\mu}C^a_{\nu]}\right) + \gamma b\,, \tag{4}$$

where the notation $f([m])$ means that the function f depends on the variable m and its subsequent derivatives up to a finite order. If a has a fixed, finite ghost number, then a can only contain a finite number of antifields. If we assume in addition that a has a bounded number of derivatives, as we shall do from now on, then, the α_J are polynomials.

In the sequel, the polynomials $\alpha_J\left([K],[h^*],[C^*]\right)$ in the linearized curvature $K^a_{\alpha\beta\mu\nu}$, the antifields $h^{*\mu\nu}_a$ and $C^{*\mu}_a$, as well as all their derivatives, will be called "invariant polynomials". They may of course have an extra, unwritten, dependence on dx^μ, i.e., be exterior forms. At zero antighost number, the invariant polynomials are the polynomials in the linearized curvature $K^a_{\alpha\beta\mu\nu}$ and its derivatives.

We shall need the following theorem on the cohomology of d in the space of invariant polynomials.

Theorem 1 *In form degree less than* n *and in antighost number strictly greater than* 0*, the cohomology of* d *is trivial in the space of invariant polynomials.*

That is to say, if α is an invariant polynomial with $antigh(\alpha) > 0$, the equation $d\alpha = 0$ implies $\alpha = d\beta$ where β is also an invariant polynomial. For the proof, see [1].

4 Cohomology of δ modulo d

The next cohomology that we shall need is $H(\delta|d)$ in the space of local forms that do not involve the ghosts ($H(\delta|d)$ is trivial in the space of forms with positive ghost number [8]). This cohomology has an interesting interpretation in terms of conservation laws ([9] for more information).

The following vanishing theorems can be proven:

Theorem 2 *The cohomology groups* $H^n_p(\delta|d)$ *vanish in antighost number strictly greater than* 2*,*

$$H^n_p(\delta|d) = 0 \ \text{ for } p > 2. \tag{5}$$

The proof of this theorem is given in [9] and follows from the fact that linearized gravity is a linear, irreducible, gauge theory.

In antighost number two, the cohomology is also completely known,

Theorem 3 *A complete set of representatives of $H^n_2(\delta|d)$ is given by the antifields $C^{*\mu}_a$ conjugate to the ghosts, i.e.,*

$$\delta a^n_2 + da^{n-1}_1 = 0 \Rightarrow a^n_2 = \lambda^a_\mu C^{*\mu}_a dx^0 dx^1 \cdots dx^{n-1} + \delta b^n_3 + db^{n-1}_2 \tag{6}$$

where the λ^a_μ are constant.

For the proof, see [1].

We have discussed so far the cohomology of δ modulo d in the space of arbitary functions of the fields $h^a_{\mu\nu}$, the antifields, and their derivatives. One can also study $H^n_k(\delta|d)$ in the space of invariant polynomials in these variables, which involve $h^a_{\mu\nu}$ and its derivatives only through the linearized Riemann tensor and its derivatives (as well as the antifields and their derivatives). The above theorems remain unchanged in this space. This is a consequence of

Theorem 4 *Let a be an invariant polynomial. Assume that a is δ trivial modulo d in the space of all (invariant and non-invariant) polynomials, $a = \delta b + dc$. Then, a is also δ trivial modulo d in the space of invariant polynomials, i.e., one can assume without loss of generality that b and c are invariant polynomials.*

The proof is given in [1].

5 Construction of the general gauge theory of interacting gravitons by means of cohomological techniques

To compute $H^{n,0}(s|d)$, we shall use an expansion according to the antighost number, as in [10]. Let a be a solution of $sa + db = 0$ with ghost number zero. One can expand a as $a = a_0 + a_1 + \cdots a_k$ where a_i has antighost number i (and ghost number zero). Without loss of generality, one can assume that this expansion stops at some finite value of the antighost number. This was shown in [10] (section 3), under the sole assumption that the first-order deformation of the Lagrangian a_0 has a finite (but otherwise arbitrary) derivative order.

The previous theorems on the characteristic cohomology imply that one can remove all components of a with antighost number greater than or equal to 3. Indeed, the (invariant) characteristic cohomology in degree k measures precisely the obstruction for removing from a the term a_k of antighost number k (see [1]). Since $H^n_k(\delta|d)$ vanishes for $k>2$, one can assume $a = a_0 + a_1 + a_2$ and $b = b_0+b_1$ [1]. Inserting these expressions in $sa+db=0$, we get $\delta a_1+\gamma a_0 = db_0$, $\delta a_2+\gamma a_1 = db_1$ and $\gamma a_2 = 0$. Let us recall the meaning of the various terms in a : a_0 is the deformation of the lagrangian; a_1 captures the information about the deformation of the gauge transformations; while a_2 contains the information about the deformation of the gauge algebra.

5.1 Determination of a_2

As we have seen in section 3, the general solution of $\gamma a_2 = 0$ reads, modulo trivial terms, $a_2 = \sum_J \alpha_J \omega^J$, where the α_J are invariant polynomials (see (4)). A necessary (but not sufficient) condition for a_2 to be also a solution of $\delta a_2 + \gamma a_1 + db_1 = 0$, so that a_1 exists, is that α_J be a non trivial element of $H^n_2(\delta|d)$ [1]. Thus, the polynomials α_J must be linear combinations of the antighosts $C^*_{\alpha a}$. The monomials ω^J have ghost number two; so they can be of only three possible types, namely, $C^a_\alpha C^b_\beta$, $C^a_\alpha \partial_{[\beta} C^b_{\gamma]}$ and $\partial_{[\alpha} C^a_{\beta]} \partial_{[\gamma} C^b_{\delta]}$. They should be combined with C^{*a}_α to form a_2. By Poincaré (and PT) invariance, the only possibility is to take $C^a_\alpha \partial_{[\beta} C^b_{\gamma]}$, which yields $a'_2 = -C^{*\beta}_a C^{\alpha b} \partial_{[\alpha} C^c_{\beta]} a^a_{bc}$. Notice that the constants a^a_{bc} are introduced here as the constants on which the general solution a_2 depends.

The a^a_{bc} can be identified with the structure constants of a N-dimensional algebra $\mathcal{A}$. Let V be an "internal" vector space of dimension N; we define a product in V through

$$(x \cdot y)^a = a^a_{bc} x^b y^c, \quad \forall x, y \in V. \tag{7}$$

The vector space V equipped with this product defines the algebra $\mathcal{A}$. At this stage, $\mathcal{A}$ has no particular further structure. Extra conditions will arise, however, from the demand that a (and not just a_2) exists and defines a deformation that can be continued to all orders. We shall recover in this manner the conditions found in [3], plus one additional condition that will play a crucial role.

We redefine a_2 by adding a γ–exact term to a'_2, in order to make the subsequent calculations simpler:

$$a_2 = C^{*\beta}_a C^{\alpha b} \partial_\beta C^c_\alpha a^a_{bc} = a'_2 + \gamma \left(\frac{1}{2} C^{*\beta}_a C^{\alpha b} h^c_{\alpha\beta} a^a_{bc} \right) . \tag{8}$$

In terms of the algebra of the gauge transformations, this term a_2 implies that the gauge parameter $\zeta^{a\mu}$ corresponding to the commutator of two gauge transformations with parameters $\xi^{a\mu}$ and $\eta^{a\mu}$ is given by

$$\zeta^{a\mu} = a^a_{bc}[\xi^b, \eta^c]^\mu \tag{9}$$

where $[,]$ is the Lie bracket of vector fields. It is worth noting that at this stage, we have not used any a priori restriction on the number of derivatives (except that it is finite). The assumption that the interactions contain at most two derivatives will only be needed below. Thus, the fact that a stops at a_2, and that a_2 is given by (8) is quite general.

Differently put: to first-order in the coupling constant, the deformation of the algebra of the spin-2 gauge symmetries is universal and given by (8). There is no other possibility.

5.2 Determination of a_1

In order to find a_1 we have to solve the equation $\delta a_2 + \gamma a_1 = db_1$. As shown in [1], this equation for a_1 has a solution if and only if

$$a^a_{bc} = a^a_{(bc)}\ , \tag{10}$$

so that the algebra $\mathcal{A}$ defined by the a^a_{bc}'s must be commutative. This result is not surprising in view of the form of the commutator of two gauge transformations since (9) ought to be antisymmetric in ξ^a and η^a. When (10) holds, a_1 is given by $a_1 = -h^{*\beta\gamma}_a C^{\alpha b}\left(\partial_\gamma h^c_{\alpha\beta} + \partial_\beta h^c_{\alpha\gamma} - \partial_\alpha h^c_{\gamma\beta}\right) a^a_{bc}$ up to a solution of the "homogenous" equation $\gamma a_1 + db_1 = 0$.

The solutions of the homogeneous equation do not modify the gauge algebra (since they have a vanishing a_2), but they do modify the gauge transformations. However, they involve too many derivatives (see [1]) and so, are excluded by our number-of-derivatrives-assumption.

5.3 Determination of a_0

We now turn to the determination of a_0, that is, to the determination of the deformed lagrangian at first order in g. The equation for a_0 is $\delta a_1 + \gamma a_0 = db_0$. It is shown in [1] that this equation for a_0 has a solution if and only if

$$a_{abc} = a_{(abc)}. \tag{11}$$

An algebra which fulfills $a_{abc} = a_{cba}$ is called hilbertian, or, in the real case considered here, "symmetric". When (11) holds, a_0 exists and is given by a cubic expression whose explicit form may be found in [1]. We have therefore proven that a gauge theory of interacting spin two fields, with a non trivial gauge algebra, is first-order consistent if and only if the algebra $\mathcal{A}$ defined by a^a_{bc}, which characterizes a_2, is commutative and symmetric.

Again, there is some ambiguity in a_0 since we can add to it any solution of the "homogeneous" equation $\gamma\tilde{a}_0 + d\tilde{b}_0 = 0$ without a_1. If one requires that $\tilde{a}_0$ has no more than two derivatives - as done here -, there is only one possibility, namely $-2\tilde{\Lambda}^{(1)}_a h^{a\mu}_\mu$ where the $\tilde{\Lambda}^{(1)}_a$'s are constant. This term fulfills $\gamma\tilde{\Lambda}^{(1)}_a h^{a\mu}_\mu = \partial_\mu(2\tilde{\Lambda}^{(1)}_a \epsilon^{a\mu})$ and is of course the (linearized) cosmological term. There is no other term [1].

5.4 The associativity of the algebra from the absence of obstructions at second order

The master equation at order two is $(W_1, W_1) = -2sW_2$. Given $W_1 = \int d^n x\ (a_0 + a_1 + a_2)$, W_2 exists if and only if (W_1, W_1) is BRST-exact. This happens if and only if the a^a_{bc} fulfill [1]

$$a^a_{d[b} a^d_{f]c} = 0\,, \tag{12}$$

which is the associative property for the algebra $\mathcal{A}$ defined by the a^a_{bc}. Thus, $\mathcal{A}$ must be commutative, symmetric and associative.

6 Impossibility of cross-interactions

Finite-dimensional algebras that are commutative, symmetric and associative have a trivial structure: they are the direct sum of one-dimensional ideals.

To see this, one proceeds as follows. The algebra operation allows us to associate to every element of the algebra $u \in \mathcal{A}$ a linear operator $A(u) : \mathcal{A} \longrightarrow \mathcal{A}$ defined by $A(u)v \equiv u \cdot v$. In a basis $(e_1, \ldots, e_m)$, one has $v = v^a e_a$ and $A(u)^c{}_b = u^a a^c_{ab}$. Because of the associative property, the operators $A(u)$ provide a representation of the algebra $A(u)A(v) = A(u \cdot v)$ and so, since the algebra is commutative, $[A(u), A(v)] = 0$.

Now, the free Lagrangian defines a scalar product in the algebra, $(u, v) = \delta_{ab} u^a v^b$. The symmetry property $a_{abc} = a_{(abc)}$ expresses that the operators $A(u)$ are all symmetric $(u, A(v)w) = (A(v)u, w)$, that is, $A(u) = A(u)^T$. Then the operators $A(u)$, $u \in \mathcal{A}$ are diagonalizable by a rotation. Since they are commuting, they are simultaneously diagonalizable. In a basis $\{e_1, \ldots, e_m\}$

in which they are all diagonal, one has $A(e_a)e_b = \alpha(a,b)e_b$ for some numbers $\alpha(a,b)$ and thus $e_a \cdot e_b = A(e_a)e_b = \alpha(a,b)e_b = e_b \cdot e_a = A(e_b)e_a = \alpha(b,a)e_a$. So $\alpha(a,b) = 0$ unless $a = b$.

Consequently, the structure constants a^a_{bc} of the algebra $\mathcal{A}$ vanish whenever two indices are different. There is no term in W_1 coupling the various spin-2 sectors, which are therefore completely decoupled. Only self-interactions are possible. The first-order deformation W_1 is in fact the sum of Einstein cubic vertices (one for each spin-2 field with $\alpha(a,a) \neq 0$) + (first-order) cosmological terms.

Once the absence of cross-interactions is established, it is easy to show that the full Lagrangian is given by the sum (2) of Einstein actions, which are known to be solutions of the deformation problem. The discussion may be found in [1].

7 Conclusions

In this note, we have reported a no-go result on cross-interactions between a collection of massless spin-2 fields. Our method relies on the antifield approach and uses cohomological techniques.

Although we restricted the discussion to interactions with at most two derivatives, the same conclusion seems to hold in general, except for obvious cross-interactions involving the linearized curvatures, which do not change the gauge transformations (see [1] for comments on this). Also, standard matter is not expected to alter the discussion (the case of scalar matter is considered explicitly in [1]). The interacting theory describes thus parallel worlds, and, in any given world, there is only one massless spin-2 field. This massless spin-2 field has the standard graviton couplings with the fields living in its world (including itself), in agreement with the single massless spin-2 field studies of [11].

The fact that that there is effectively only one type of gravitons is therefore not a choice but a necessity that adds to its great theoretical appeal. This feature is one of the arguments used to rule out $N > 8$ extended supergravity theories, since these would involve gravitons of different types (besides particles of spin greater than 2, whose coupling to gravity is known to be problematic). Our no-go theorem extends the analysis of [12], where the coupling of one massless spin-2 field to gravity described by Riemannian geometry was shown to be problematic.

We close by noting that one key assumption underlying our negative result is the presence of only a finite number of gravitons. This assumption was crucially used in showing that the structure of the algebra $\mathcal{A}$ was trivial. If this assumption is relaxed, cross-interactions become possible [13].

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